increased. At the sideline, the peak scales as a Strouhal number. In this case (see Fig. 20 of Lush[8]), the peak Strouhal numbers lay between 0.75 and 0.93 at $\theta = 90$ deg. Below a Reynolds number of about 10^5, the peak Strouhal number is independent of emission angle and falls in the range of 0.15 to 0.3.

According to McCartney,[9] the peak frequencies of self and shear noise spectra can be expressed in terms of the derivatives of a Fourier transform of the second-order moving frame correlation tensor of turbulent velocities at two points at different times. He further proposed that if the space and time dependence of the correlation tensor can be separated, the peak frequency can be calculated for any given time decay of the correlation tensor. It follows that if the observed shift in peak frequency with Reynolds number is an intrinsic property of jet noise, the decaying profile of the second-order moving frame correlation tensor of turbulent velocities would also be dependent on Reynolds number. No attempt was made to confirm this hypothesis experimentally during this study.

It is difficult to quantify the critical value of 10^5 for Reynolds number. Apparently the criterion is one of whether or not transition to turbulence occurs in the nozzle boundary layer or in the initial free shear layer. Presumably, the jet noise characteristics stabilize when the nozzle boundary layer is turbulent. This is supported by the fact that crude calculations indicate that transition to turbulence occurs in the nozzle boundary layer at Reynolds numbers higher than about 10^5.

McLaughlin et al.[4] found that a few discrete modes at a Strouhal number of about 0.2 in a low Reynolds number, supersonic jet are powerful noise generators. This compares favorably with the present study in which the peak Strouhal number was also found to be of order 0.2. The detailed information reported herein would not be uncovered in octave and third octave analysis of the noise signal. Apparently, narrow band spectral analysis is necessary to unveil certain details of jet noise which have received scant attention in the literature.

Acknowledgment

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Numerical Solution for Perpendicular Sonic Hydrogen Injection into a Ducted Supersonic Airstream

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Nomenclature

c_v = specific heat at constant volume
e = total internal energy per unit volume
f = hydrogen mass fraction
p = static pressure
T = static temperature
u = streamwise velocity
v = normal velocity
x = streamwise coordinate
y = normal coordinate
Le = Lewis number
Pr = Prandtl number
Sc = Schmidt number
μ = total viscosity

Introduction

SCRAMJET engine concepts being investigated at the NASA Langley Research Center[1] incorporate both parallel and perpendicular hydrogen fuel injection. This approach tailors the injection to an optimum heat release schedule in the engine over a flight Mach number range. Efficient engine design requires a detailed understanding of the flowfield near each type fuel injector. In order to limit the broad area of injector design that must be considered experimentally, an analytical tool is needed to survey a number of attractive options. This Note discusses a computer program being developed to study the flowfield near opposing perpendicular fuel injectors and presents some preliminary results used during the initial evaluation of the code.

Analysis

The flowfield near a perpendicular fuel injector (Fig. 1) is complex, and presents a very challenging problem for numerical analysis. Regions of separation and recirculation exist both in front of and behind the jet, and the flow undergoes severe turning through a bow shock upstream of and an expansion behind the jet. The analysis of such a flow requires the solution of the full (elliptic) compressible Navier-Stokes, energy and species equations. In conservative form, these equations in two dimensions are[2,3]

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \tag{1a}$$

$$U = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ e \\ \rho f \end{Bmatrix} \tag{1b}$$

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$$F=\left\{\begin{array}{l}\rho u\\ \rho u^2+\sigma_x\\ \rho uv+\tau_{xy}\\ (e+\sigma_x)u+\tau_{yx}v-\kappa\dfrac{\partial e_s}{\partial x}\\ \rho vf-\Gamma\dfrac{\partial f}{\partial x}\end{array}\right\} \tag{1c}$$

$$G=\left\{\begin{array}{l}\rho v\\ \rho uv+\tau_{yx}\\ \rho v^2+\sigma_y\\ (e+\sigma_y)v+\tau_{xy}u-\kappa\dfrac{\partial e_s}{\partial y}\\ \rho vf-\Gamma\dfrac{\partial f}{\partial y}\end{array}\right\} \tag{1d}$$

where

$$\sigma_x=p-\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-2\mu\frac{\partial u}{\partial x},\quad \tau_{xy}=\tau_{yx}=-\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)$$

$$\sigma_y=p-\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-2\mu\frac{\partial v}{\partial y},\quad \lambda=-\frac{2}{3}\mu$$

$$\kappa=\frac{\gamma\mu}{Pr},\quad \Gamma=\frac{\mu}{Sc}=\frac{\mu}{Pr}\quad (Le=1),\quad e_s=c_vT$$

The two-dimensional form was chosen for initial studies of the injector near field due to the prohibitively large computational domain required for three dimensions. In two dimensions, the problem is tractable on today's computers and allows for an initial study of injection phenomena while still providing for the development of techniques to be used later in a three-dimensional analysis.

Equations (1) were solved using the MacCormack time-split, finite-difference relaxation technique.[3] Using this technique, a program (TWODLE, two-dimensional elliptic) was developed to consider the turbulent nonreacting flow of hydrogen and air in a rectangular duct. Turbulence was modeled using an algebraic eddy viscosity approach[4] and the Reynolds analogy. Boundary conditions of the reflection class[3] were employed along the walls. The program was then applied to a model problem representing a two-dimensional equivalent of the flow between two struts in the scramjet engine (Fig. 1). Inlet air is introduced into a duct 6 cm high by 10-cm long at $M=2.7$ and 800 K. Gaseous hydrogen is injected perpendicular to the air at $M=1.1$ and 242 K from two opposing 0.18-cm-wide side wall slots. The static pressure ratio between the hydrogen jet and the inlet air is 10. The wall boundary conditions are set to require zero velocity components and zero heat flux except at the slots where the velocity and temperature are fixed at their initial values. The normal derivative of the hydrogen mass fraction is required to vanish at the wall, consistent with the no-flux condition there. These boundary conditions are applied following each predictor and corrector step in the integrator. The density at the jet exit plane along the wall is fixed; elsewhere the wall density is set equal to the nodes just off the wall, the lower wall being set only during the predictor step and the upper wall only during the corrector step as the continuity equation is spatially first order. At the inflow boundary, all dependent variables are held at their initial values; at the outflow boundary, each dependent variable is determined by extrapolation from upstream values. The pressure is calculated from the equation of state, and bouyancy effects are assumed negligible.

A fourth-order numerical damping term[3] was employed to reduce numerical oscillations in the neighborhood of shocks. This term was modified from its original form to be proportional not only to the second derivative of pressure, but also the second derivative of temperature, to counter a numerical problem at the reattachment shock downstream of the slot. This was necessary because oscillations in the density and temperature differed in phase and offset each other to produce a reasonably uniform pressure derivative; thus, inadequate smoothing was present to allow convergence. The oscillation near the reattachment point was therefore detected and smoothed by also considering the temperature field.

Results

The governing equations were applied to the model problem described above, and they were relaxed in time until steady-state conditions were achieved. The duct was spanned by a grid 60 nodes high by 40 nodes long, geometrically stretched away from the walls and the slots. The resulting velocity vector field near the lower injector is shown in Fig. 2. Note the presence of a small recirculating region upstream of the slot and a large recirculating region behind the slot that persists well downstream. Note also the presence of a bow shock in front of the slot producing significant turning over the hydrogen jet. The reattachment point terminating the downstream recirculating region is also evident. The separated region upstream of the slot is smaller than expected, and it is likely being underestimated because of the limited nature of the turbulence model in recirculating flows.

Figure 3 describes the resulting steady-state hydrogen mass fraction contours in the lower half duct. With a static jet to freestream pressure ratio of 10, the penetration of the hydrogen jet into air cross stream is significant. The 1% H_2

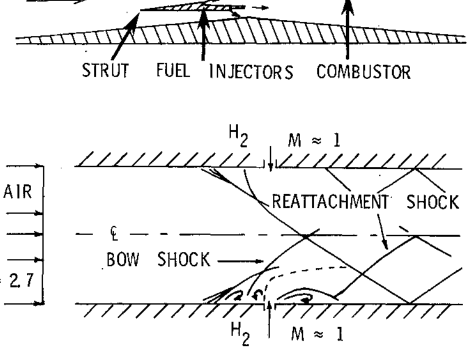

Fig. 1 Cross-section schematic of scramjet and model problem for flow between two struts.

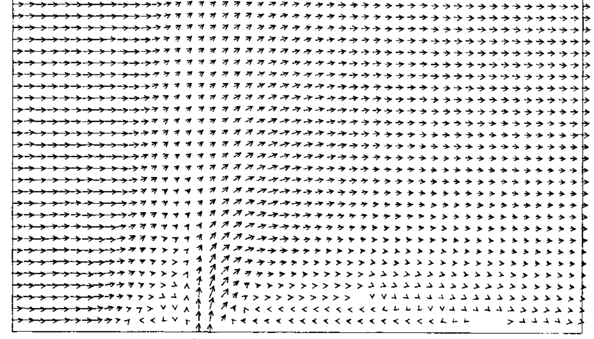

Fig. 2 Velocity vector field near lower injector.

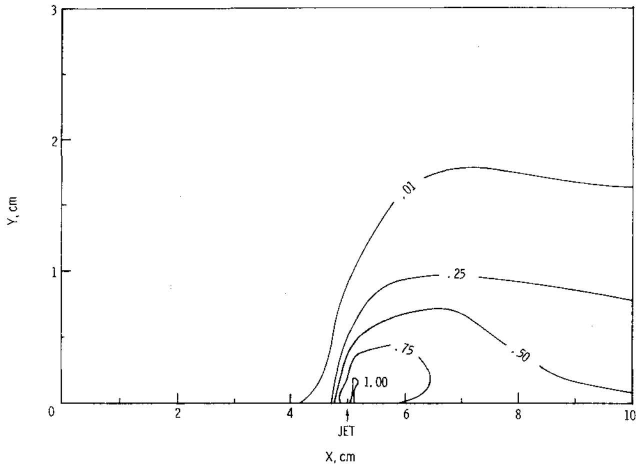

Fig. 3 Hydrogen mass fraction contours in lower half duct.

contour line lies at ¼ the duct height after proceeding only 5 slot widths downstream. The recirculating region upstream of the slot convects hydrogen forward of the jet, and the mixture goes from fuel lean to rich as the slot is approached. The separated flow downstream of the slot captures hydrogen, producing a very fuel rich region. These are phenomena that have been observed in experimental scramjet design, the upstream region providing a beneficial area for fuel ignition and the downstream region (being fuel rich) imposing an ignition problem.

Conclusions

A program has been developed to simulate the near sonic perpendicular injection of hydrogen into a ducted supersonic air stream. A case using actual conditions encountered in current scramjet design has been successfully analyzed, producing results that agree qualitatively with experimental observations. A modified form of a popular numerical damping scheme was employed to prevent numerical oscillations near shocks. The damping term was made proportional not only to the second derivative of pressure, but also the second derivative of temperature to produce a stable solution behind the hydrogen jet in the neighborhood of the recompression shock. Additionally, alternating boundary conditions were used to specify the wall density yielding a more physically realistic density field near the wall. A number of improvements and extensions still remain, however, none of which are trivial. First, the turbulence model must be improved to more adequately consider the complex flowfield that exists. Second, a chemistry model must be added to consider at least the H_2-air reaction that occurs; more complete finite rate modeling will likely require an implicit rather than explicit integrator. Finally the actual problem that must be considered is three- rather than two-dimensional, limiting implementation of this work to all but the largest current or future vector computers. The current effort does, however, represent a first attempt to numerically model the perpendicular fuel injector problem in the engine, and should provide guidance in the on going scramjet design effort at Langley Research Center.

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Asymmetry of a Circular Jet Observed in Near and Far Fields

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Introduction

An extensive survey of the turbulence characteristics and the acoustic field of low Reynolds number jets has been recently completed. One of the unexpected findings was that slight but detectable asymmetries exist in both the flowfield and the surrounding acoustic field.

During preliminary testing, the true jet axis was determined with a single hot-film probe. The aerodynamic jet axis is defined as the locus of points in the flow at which the turbulence intensity is a minimum. Measurements were made at 10 equally spaced X stations between $X=D$ and $X=10D$ from the jet exit plane. The results show that at each X station the deviation from the geometrical jet axis was on the order of 2-4% of the nozzle diameter, and the true jet axis has the form of a helical wave. This helical wavelike jet axis was observed in three different jets of 12.7, 6.35, and 3.18-mm diameter. It was further confirmed that rotating of the nozzle or settling chamber does not give any noticeable effect on the profile of the true jet axis and that this profile changed significantly from one day to another. This may indicate that the helical wavelike disturbances are generated far upstream of the nozzle. One may speculate that since the turbulence is more sensitive to the variations of flow conditions than is the mean flow, the asymmetry of jet flow is more easily observed in turbulence intensity measurements. In fact, this was clearly confirmed in the present investigation. It is interesting to note that the asymmetry of the jet flow was observed not only in the flowfield but also in the radiated field. It should be added that the circumferential asymmetry of the sound pressure level was more strongly observed in the sideline noise signature than at observation angles close to the jet axis.

Measurements

Turbulence intensity measurements were made in three different sized jets at various Mach numbers. Figure 1 contains an example of the resulting data obtained in a 12.7-mm jet at a Mach number, M, of 0.297. These data represent the measured axial component of turbulence measured at various axial and radial positions. A single hot-film probe was traversed radially across the jet at distances of $Z=\pm 3D$ from the centerline. The measurements were made at equally spaced axial stations ranging from 1-10 diameters from the plane of the nozzle.

The turbulence intensity attains a maximum value near $Z/D=0.5$ at any downstream station. The asymmetry of the profiles is evident. This asymmetry of the jet flow is not due to the nozzle configuration. This was confirmed by rotating the nozzle by 180 deg, yielding the same results. It is debatable whether these results imply a natural asymmetry of the flow due to a helical mode in the jet turbulence or upstream flow

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